Three Dimensional Object Representation

By
Poonam Saini
Dept. of Computer Science & Engineering
Sir Padampat Singhania University
Udaipur

3D Object Representation

- Graphics scenes can contains many different kinds of objects like tree, flowers, clouds, rocks, water, rubber, paper, marbel, steel, glass etc.
- To produce realistic displays of scenes, representation are used to accurately model object characteristics.
- Polygon and quadratic surfaces provides descriptions ellipsoids etc.
- Spline surfaces and construction techniques are useful for designing aircraft wings, gears and other engineering structures with curved surfaces.
- Procedural methods gives representations for clouds, clumps of grass etc.

Representation Schemes for solid Objects

- **Boundary Representation(B-reps):** It describe a 3D object as a set of surfaces that separate the object interior from the environment.
- **Space-partitioning representation:** These are used to describe interior properties by partitioning the special regions containing an object into a set of small, non overlapping contiguous solids.

Polygon surfaces

- Polygon surfaces are commonly used for boundary representation of a graphics object in 3D as they enclose the object interior.
- Many graphics systems store all object descriptions as sets of surface polygons.
- All surfaces are described with linear equations.
- Therefore, polygon descriptions are often referred to as standard graphics object.

Polygon Tables

- Polygon surface is specified with a set of vertex coordinates and associated attribute parameters.
- Information of each polygon is placed into tables which are used for subsequent processing displays and manipulation of an object.
- Polygon data tables can be organized into two groups:

  → **Geometric Table:** It contains vertex coordinates and parameters to identify the spatial orientation of the polygon surface.

  → **Attribute Table:** It contains the attributes information of an object that include degree of transparency, its surface reflectivity and texture characteristics.

Geometric Tables

- These store three lists

→ Vertex table: coordinates values for each vertex is stored.

→ Edge table: It contains pointers back into the vertex table to identify the vertices for each polygon edge.

→ Polygon table: It contains pointers back into the edge table to identify the edges for each polygon.

Geometric Tables Example

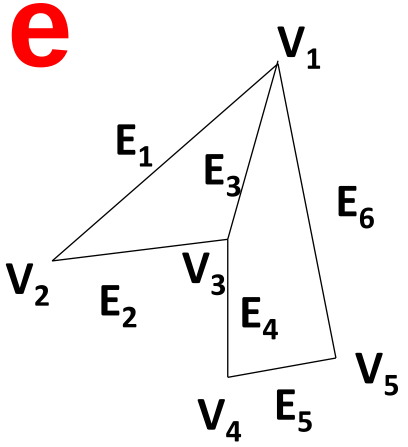

Fig: Tow adjacent polygons on an object surface.

Vertex Table
V_1 : x1, y1, z1
V_2 : x2, y2, z2
V_3 : x3, y3, z3
V_4 : x4, y4, z4
V_5 : x5, y5, z5

Edge Table
$E_1 : V_1 , V_2$
$E_2 : V_2 , V_3$
$E_3 : V_3 , V_1$
$E_4 : V_3 , V_4$
$E_5 : V_4 , V_5$
$E_6 : V_5 , V_1$

Polygon Surface Table
$S_1 : E_1 , E_2 , E_3$
$S_2 : E_3 , E_4 , E_5 , E_6$

Polygon Meshes

- Sometimes, surfaces are tiled to produce the polygon mesh. For eg.: The surface of cylinder is represented as a polygon mesh.
- Polygon mesh approximation to a curved surface can be improved by dividing the surface into smaller polygon facets.
- One type of polygon mesh is triangle strip. This function produces n-2 connected triangles

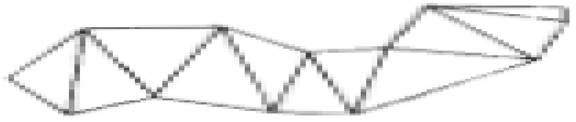

Figure 10-6
A triangle strip formed with 11 triangles connecting 13 vertices.

- Quadrilateral mesh generates a mesh of (n-1)/(m-1) quadrilaterals given the coordinates for an n x m

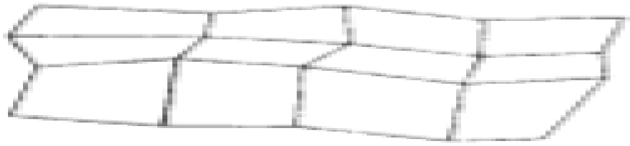

Figure 10-7
A quadrilateral mesh containing 12 quadrilaterals constructed from a 5 by 4 input vertex array.

Curved lines and surfaces

- Displays of 3D curved lines and surfaces can be generated from an input set of mathematical functions defining the objects or from a set of user specified data points.
- When functions are specified, a package can project the defining equations for a curve to display plane and plot pixel positions along the path of the projected function.
- For surfaces, a functional description is often selected to produce a polygon mesh approximation to the surface.
- Usually, this is done with triangular polygon patches to ensure that all vertices of any polygon are in one plane

Quadric surfaces

- Quadric surfaces are described with second degree equations. They include spheres, ellipsoids, paraboloids, hyperboloids.
- Spheres and ellipsoids are the common elements of graphics scenes.

Sphere

- A spherical surface with radius r, centered on the coordinate origin is defined as the set of points(x,y,z) that satisfy the equation.

$$x^2 + y^2 + z^2 = r^2$$

$$x = r\cos\phi\cos\theta, \quad -\pi/2 \le \phi \le \pi/2$$

$$y = r\cos\phi\sin\theta, \quad -\pi \le \theta \le \pi$$

$$z = r\sin\phi$$

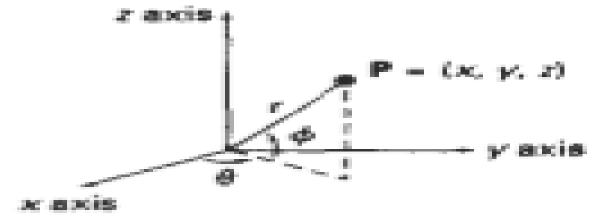

Figure 10-8
Parametric coordinate position (r, θ, ϕ) on the surface of a sphere with radius r.

Quadric surfaces

Ellipsoid

An ellipsoid surface is an extension of a spherical surface where the radii in the three mutually perpendicular directions can have different values.

The Cartesian representation for the points over the Surface of an ellipsoid centered on the origin is:

$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

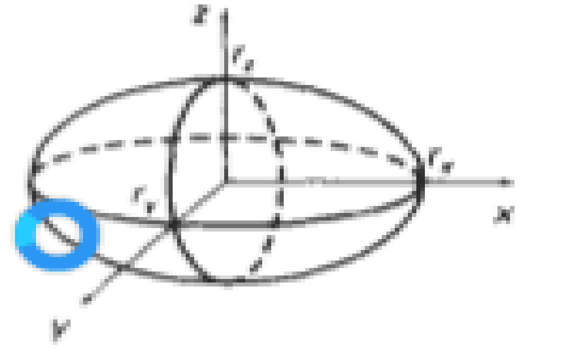

Figure 10-10
An ellipsoid with radii r_x, r_y, and r_z centered on the coordinate origin.

A parametric representation for an ellipsoid in terms Of latitude angle Φ and longitude angle Θ.

$$\begin{aligned} x &= r_x \cos\phi \cos\theta, & -\pi/2 \le \phi \le \pi/2 \\ y &= r_y \cos\phi \sin\theta, & -\pi \le \theta \le \pi \\ z &= r_z \sin\phi \end{aligned}$$

Quadric surfaces

Torus

- A torus is a doughnut-shaped object as shown in Fig.
- It can be generated by rotating a circle or other conic about a specified axis.
- The Cartesian representation for points over the surface of a torus can be written in the form

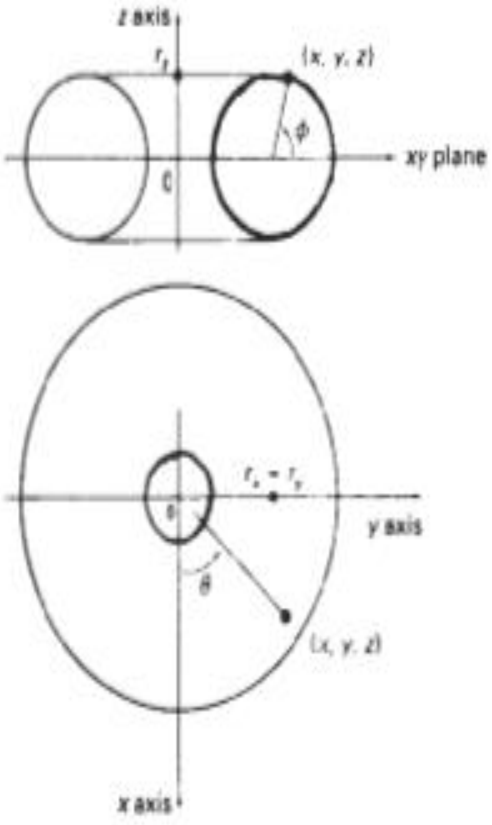

Figure 10-11
A torus with a circular cross section centered on the coordinate origin.

$$\left[r - \sqrt{\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2}\right]^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

Where r is any offset value

Quadric surfaces

Blobby Objects

- Some objects do not maintain a fixed shape , but change their surface characteristics in certain motions or when in proximity to other objects.
- Examples in this class of objects include molecular structures, water droplets and other liquid effects, melting objects and muscle shapes in human body.

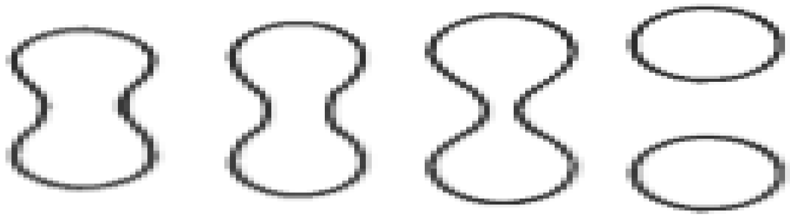

Figure 10-14
Molecular bonding. As two molecules move away from each other, the surface shapes stretch, snap, and finally contract into spheres.

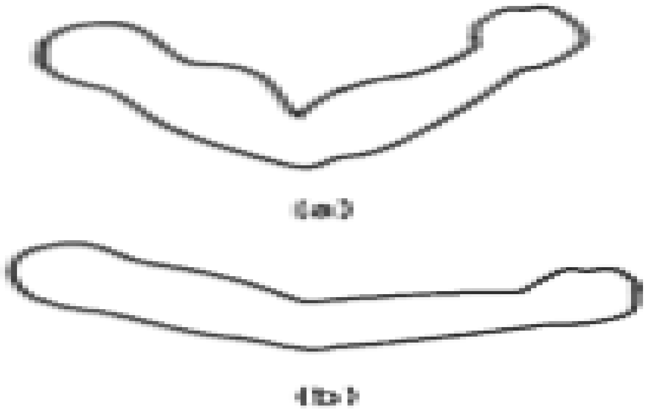

Figure 10-15
Blobby muscle shapes in a human arm.

Spline Representations

- Spline is a flexible strip used to produce a smooth curve through a designated set of points.
- Several small weights are distributed along the length of the strip to hold it in position on the drafting table as the curve is drawn.
- In computer graphics, spline curve is referred to as any composite curve formed with polynomial sections satisfying specified continuity conditions at the boundary of the pieces.
- A spline surface can be described with two sets of orthogonal spline curves.
- Splines are used to design curve and surface shapes .
- In CAD, spline include the design of automobile bodies, aircraft and spacecraft surfaces.

Interpolation and Approximation Splines

- Spline curve can be specified by giving a set of coordinate positions called control points which indicates the general shape of the curve
- These control points are then fitted with piecewise continuous parametric polynomials functions in one of the two ways:

→ When polynomial sections are fitted so that the curve passes through each control point. The resulting curve is said to interpolate the set of control points.

Figure 10-19
A set of six control points interpolated with piecewise continuous polynomial sections.

Interpolation and Approximation Splines

→When the polynomials are fitted to the general control point path without necessarily passing through any control point, the resulting curve is said to approximate the set of control points.

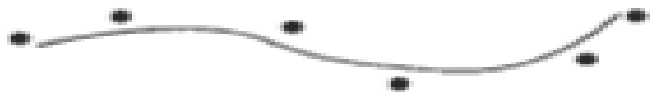

Figure 10-20
A set of six control points approximated with piecewise continuous polynomial sections

Interpolation and Approximation Splines

- Interpolation curves are commonly used to specify animation paths.
- Approximation curves are commonly used as design tools to structure object surfaces.
- The convex polygon boundary that encloses a set of control points is called the convex hull.
- Set of connected line segment is referred to as control graph of the curve.

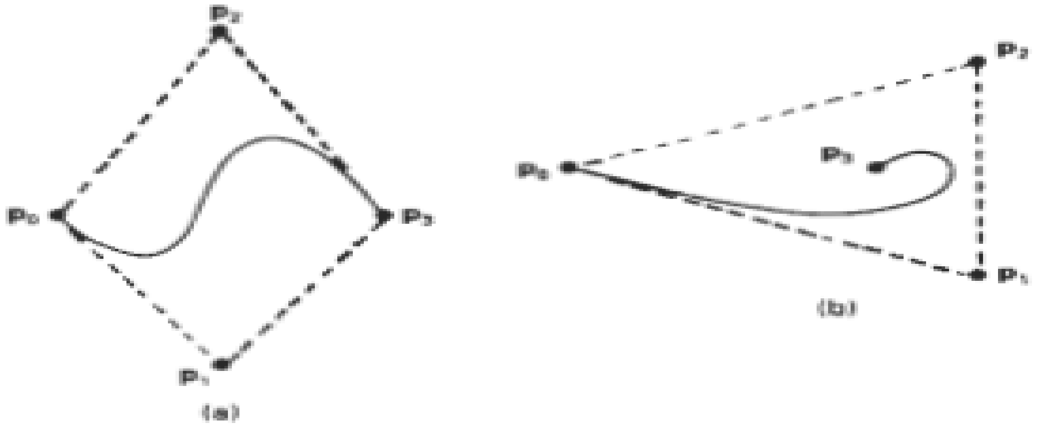

Figure 10-22
Convex-hull shapes (dashed lines) for two sets of control points.

Bezier curves & surfaces

Bezier splines have a no. of properties that make them highly useful and convenient for curve and surface design.

Bezier Curves

- Bezier curve section can be filled to any no. of control points.
- The no. of control points to be approximated and their relative positions determine the degree of the Bezier polynomial
- With the interpolation splines, a Bezier curve can be specified with boundary conditions, with a characteristic matrix or with blending functions.
- Suppose given n+1 control point positions:

  $p_k=(x_k, y_k, z_k)$ with k=0 to n
- These coordinate points can be blended to produce the following position vector p(u) that desceibes the path of an approximating Bezier polynomial function between p_0 and p_n

Bezier curves

$$\mathbf{P}(u) = \sum_{k=0}^{n} \mathbf{p}_k BEZ_{k,n}(u), \qquad 0 \le u \le 1$$

The Bézier blending functions $BEZ_{k,n}(u)$ are the *Bernstein polynomials:*

$$BEZ_{k,n}(u) = C(n, k)u^k(1-u)^{n-k}$$

where the $C(n, k)$ are the binomial coefficients:

$$C(n, k) = \frac{n!}{k!(n-k)!}$$

Equivalently, we can define Bézier blending functions with the recursive calculation

$$BEZ_{k,n}(u) = (1-u)\, BEZ_{k,n-1}(u) + u BEZ_{k-1,n-1}(u), \qquad n > k \ge 1$$

with $BEZ_{k,k} = u^k$, and $BEZ_{0,k} = (1-u)^k$. Vector equation 10-40 represents a set of three parametric equations for the individual curve coordinates:

$$x(u) = \sum_{k=0}^{n} x_k\, BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k\, BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^{n} z_k\, BEZ_{k,n}(u)$$

Bezier curves

As a rule, a Bezier curve is a polynomial of degree one less than the no. of control points used. For eg. 3 points generate a parabola, 4 points generate a cubic curve.

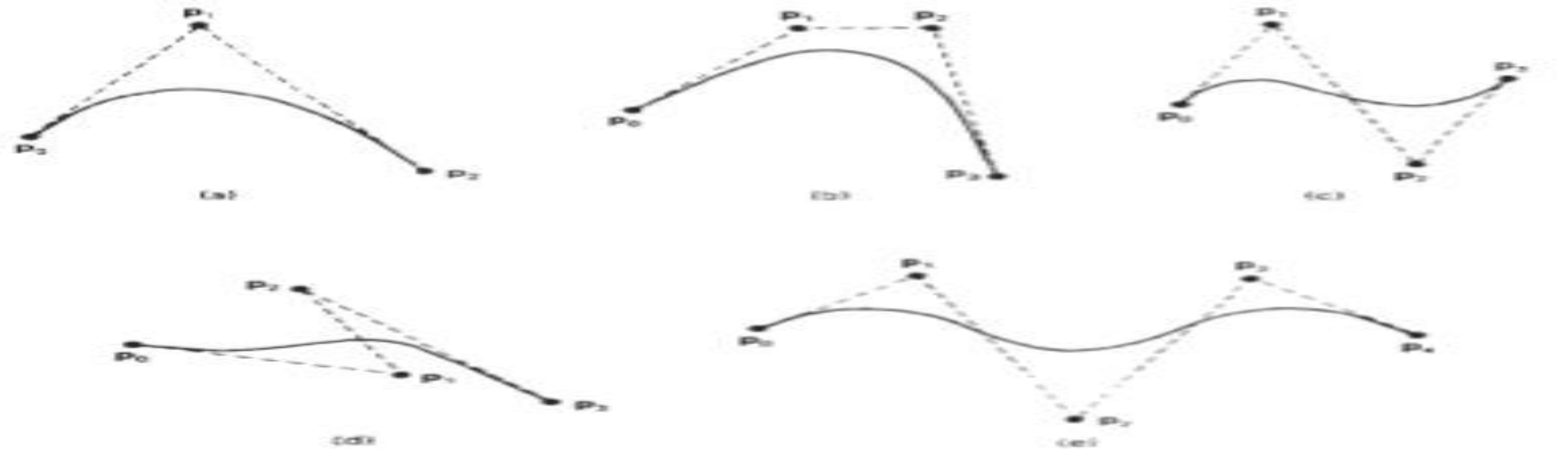

Figure 10-34
Examples of two-dimensional Bézier curves generated from three, four, and five control points. Dashed lines connect the control-point positions.

Design Techniques using Bezier curves

- Closed Bezier curves are generated by specifying the first and last control points at the same position.

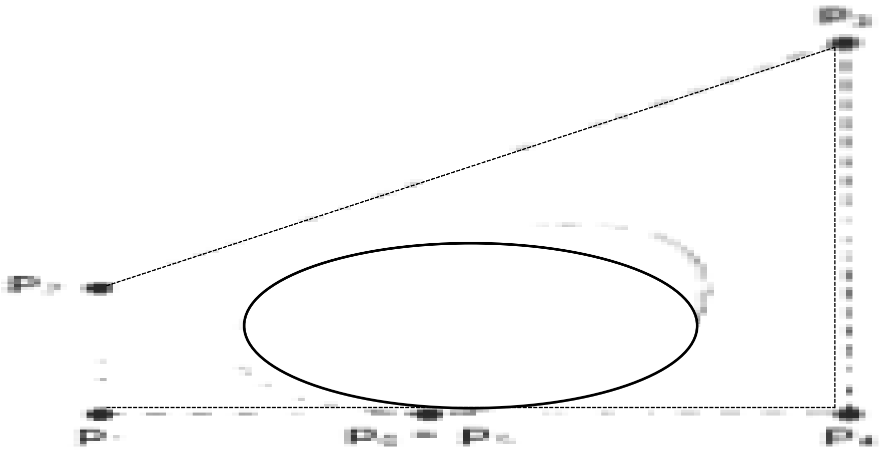

Figure 10-35
A closed Bezier curve generated by specifying the first and last control points at the same location

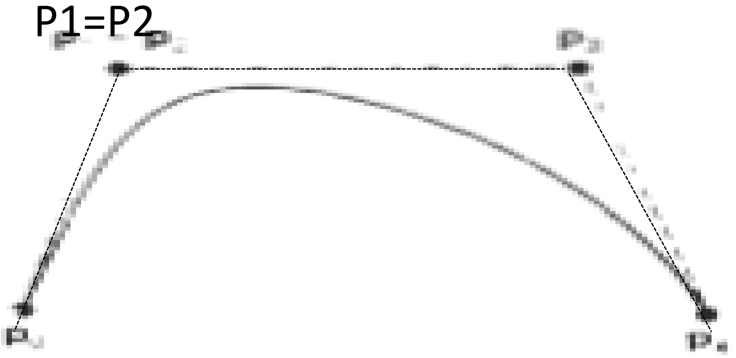

Figure 10-36
A Bezier curve can be made to pass closer to a given coordinate position by assigning multiple control points to that position

Design Techniques using Bezier curves

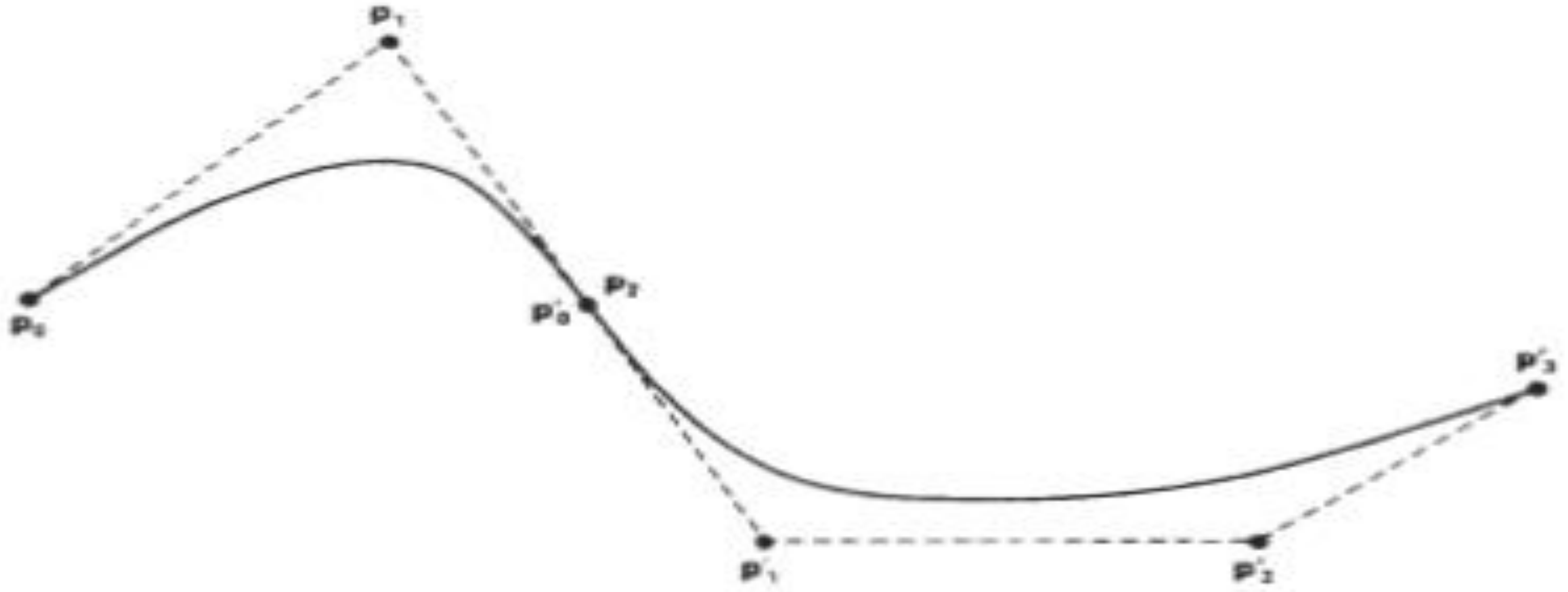

Figure 10-37
Piecewise approximation curve formed with two Bézier sections. Zero-order and first-order continuity are attained between curve sections by setting $\mathbf{p}'_0 = \mathbf{p}_2$ and by making points $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}'_1$ collinear.

B-spline curves and surfaces

Advantages of B-spline over Bezier splines

- The degree of B-spline polynomial can be set independently of the no. of control points.
- B-spline allow local control over the shape of a spline curve or surface.

But B-splines are more complex than Bezier splines.

B-spline curves

We can write a general expression for the calculation of coordinate positions along a B-spline curve in a blending-function formulation as

$$\mathbf{P}(u) = \sum_{k=0}^{n} \mathbf{p}_k B_{k,d}(u), \qquad u_{min} \le u \le u_{max}, \qquad 2 \le d \le n+1$$

B-spline curves

- Where P_k is the input set of n+1 control points.
- $B_{k,d}$ is the B-spline blending function.
- B-spline blending functions are the polynomials of degree d-1 where parameter d can be chosen to any integer value in the range from 2 up to no. of control points n+1.
- Blending function for B-spline curves are defined by Cox-deBoor recursive formula:

Where each blending function is Defined over d subintervals of the total Range of w.

$$B_{k,1}(u) = \begin{cases} 1, & \text{if } u_k \le u < u_{k+1} \\ 0, & \text{otherwise} \end{cases}$$

$$B_{k,d}(u) = \frac{u - u_k}{u_{k+d-1} - u_k} B_{k,d-1}(u) + \frac{u_{k+d} - u}{u_{k+d} - u_{k+1}} B_{k+1,d-1}(u)$$

Properties of B-spline curves

- The polynomial curve has degree $d - 1$ and C^{d-2} continuity over the range of u.
- For $n + 1$ control points, the curve is described with $n + 1$ blending functions.
- Each blending function $B_{k,d}$ is defined over d subintervals of the total range of u, starting at knot value u_k.
- The range of parameter u is divided into $n + d$ subintervals by the $n + d + 1$ values specified in the knot vector.

Cubic, Periodic B-splines

- Periodic splines are useful for generating certain closed curves.
- For example: The closed curve can be generated in sections by cyclically specifying four of the six control points.
- If any three consecutive control points are identical the curve passes through that coordinate position.

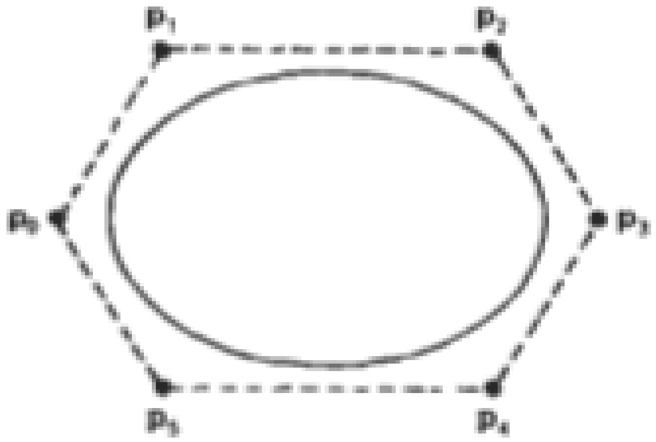

Figure 10-44
A closed, periodic, piecewise, cubic B-spline constructed with cyclic specification of the six control points.

Cubic, Periodic B-splines

For cubics, $d = 4$ and each blending function spans four subintervals of the total range of u. If we are to fit the cubic to four control points, then we could use the integer knot vector

$$\{0, 1, 2, 3, 4, 5, 6, 7\}$$

. The boundary conditions for periodic cubic B-splines with four consecutive control points, labeled $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$, are

$$\mathbf{P}(0) = \frac{1}{6}(\mathbf{p}_0 + 4\mathbf{p}_1 + \mathbf{p}_2)$$

$$\mathbf{P}(1) = \frac{1}{6}(\mathbf{p}_1 + 4\mathbf{p}_2 + \mathbf{p}_3)$$

$$\mathbf{P}'(0) = \frac{1}{2}(\mathbf{p}_2 - \mathbf{p}_0)$$

$$\mathbf{P}'(1) = \frac{1}{2}(\mathbf{p}_3 - \mathbf{p}_1)$$

Cubic, Periodic B-splines

A matrix formulation for a cubic periodic B-splines with four control points can then be written as

$$P(u) = [u^3 \; u^2 \; u \; 1] \cdot M_B \cdot \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

where the B-spline matrix for periodic cubic polynomials is

Where

$$M_B = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

Octrees

- Hierarchical structure trees called octrees are used to represent solid objects in some graphics systems.
- Medical imaging and other applications that require displays of object cross sections commonly use octree representations.
- The tree structure is organized so that each node corresponds to a region of 3D space. This representation for solids takes advantages of spatial coherence to reduce storage requirements for 3D objects.
- It also provides a convenient representation for storing information about object interiors.